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INDIAN SCHOOL AL WADI AL KABIR

Mid-term Examination (2024-25)

Sub: MATHEMATICS (041)

Date: 19-09-2024 **Set 1**

Maximum marks: 80

Class: X Time: 3 hours

	SECTION A				
Q.1.	(C) Any natural number	Q.11.	(D) mode		
Q.2.	(B) $2^2 \times 5^2$	Q.12.	(B) isosceles triangle		
Q.3.	(D) 7500	Q.13.	(D) 14		
Q.4.	(D) 0	Q.14.	$(\mathbf{A}) \frac{10}{3} cm$		
Q.5.	(C) coincident	Q.15.	(C) $x^2 - x - 12$		
Q.6.	(B) 2:7	Q.16.	$(\mathbf{D})\frac{15}{4}$		
Q.7.	(A) 2.5 cm	Q.17.	$(\mathbf{A}) \frac{2}{3}$		
Q.8.	(C) $\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	Q.18.	(B) 38		
Q.9.	(B) 7	Q.19.	(a) Both Assertion (A) and Reason (R) are true and Reason		
			(R) is the correct explanation of Assertion (A)		
Q.10.	(A) increases by 2	Q.20.	(d) Assertion (A) is false, but reason (R) is true		

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each

Q.21.

(a) Show that $3 + 2\sqrt{5}$ is an irrational number if $\sqrt{5}$ is irrational.

	→ let take that 3 + 2√5is rational number					
	→ so, we can write this answer as		½ m			
	$\Rightarrow 3 + 2\sqrt{5} = \frac{a}{b}$					
	b Here a & b use two coprime number and b ≠ 0					
	There a & b use two coprime number and b + C	½ m				
	$\Rightarrow 2\sqrt{5} = \frac{a}{-3}$					
	ь в					
	$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$ $\Rightarrow 2\sqrt{5} = \frac{a - 3b}{b}$					
	$\Rightarrow 2 \vee 5 = \frac{}{}$		½ m			
	_ a - 3b					
	$\therefore \sqrt{5} = \frac{a - 3b}{2b}$					
	a ah					
	Here a and b are integer so $\frac{a-3b}{2b}$ is a rational	l number so $\sqrt{5}$ should be	½ m			
	rational number but		72 111			
	$\sqrt{5}$ is a irrational number so it is contradict					
	TT	l				
	- Hence $3 + 2\sqrt{5}$ is irrational.	OD				
	(b) True many lights and turned on at t	OR	1da	and the athen		
	(b) Two neon lights are turned on at the same time. One blinks every 4 seconds and the other					
	blinks every 6 seconds. In 120 seconds, how many times will they blink at the same time? $4 = 2 \times 2$, $6 = 2 \times 3$ $\frac{1}{2}$ m					
	$Lcm = 2 \times 2 \times 3 = 12$ 1m					
	In 60 sec, the number of times it blinks =5					
	In 60 sec, the number of times it blinks =5 In 120 sec, the number of times it blinks =10 $\frac{1}{2}$ m					
Q.22.	Given below are three linear equations. Two of them have infinitely many solutions and two have					
Q.22.	a unique solution. State the pairs: $4x$		•			
	Show steps of comparison of ratios at			r=3.		
	Show steps of comparison of factor at	na state reason for the answe				
	4x - 5y = 3, 8x - 10y = 6	4x - 5y = 3, $5x - 4y = 3$	$5 \mid 8x - 10y$	= 6,5x - 4y = 5		
	$a1/a2 = \frac{1}{2}$, $b1/b2 = \frac{1}{2}$, $c1/c2 = \frac{1}{2}$	a1/a2 = 4/5 $b1/b2 = 5/4$	a1/a2 = 8/5	b1/b2 = 5/2		
	$a1/a2 = b1/b2 = c1/c2 = \frac{1}{2}$	a1/a2 ≠b1/b2	a1/a2 ≠b1/			
	They have infinitely many solution-	They have unique solutions	•	unique solutions		
	(1 m)	(½ m)		(½ m)		
Q.23.	Solve the quadratic equation for x :					
	$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0.$					

$\therefore \sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$	½ m	
$\Rightarrow \sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$		
$\Rightarrow \sqrt{3}x(x+\sqrt{3})+7(x+\sqrt{3})=0$	½ m	
$\Rightarrow \left(x + \sqrt{3}\right)\left(\sqrt{3}x + 7\right) = 0$	½ m	
$\Rightarrow x + \sqrt{3} = 0 \text{ or } \sqrt{3}x + 7 = 0$		
$\Rightarrow x = -\sqrt{3} \text{ or } x = -\frac{7}{\sqrt{3}} = -\frac{7\sqrt{3}}{3}$	½ m	

Q.24 (a) If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then prove that $a^2 + b^2 = m^2 + n^2$.

Given:
$$a \cos \theta + b \sin \theta = m$$
 ---(1)
 $a \sin \theta - b \cos \theta = n$ ---(2)

On squaring Eqs. (i) and (ii) and then adding the resulting equations, we get

$$m^{2} + n^{2} = (a \cos \theta + b \sin \theta)^{2} + (a \sin \theta - b \cos \theta)^{2}$$

$$= a^{2} \cos^{2} \theta + b^{2} \sin^{2} \theta$$

$$+ 2ab \sin \theta \cdot \cos \theta + a^{2} \sin^{2} \theta$$

$$+ b^{2} \cos^{2} \theta - 2ab \sin \theta \cdot \cos \theta$$

$$= a^{2}(\cos^{2} \theta + \sin^{2} \theta) + b^{2}(\sin^{2} \theta + \cos^{2} \theta)$$

$$= a^{2} + b^{2}$$

(b) Prove that:
$$\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \csc A$$
.

1 m ½ m

$=\sqrt{\frac{(\sec\theta-1)(\sec\theta-1)}{(\sec\theta+1)(\sec\theta-1)}}+\sqrt{\frac{(\sec\theta+1)(\sec\theta+1)}{(\sec\theta-1)(\sec\theta+1)}}$	½ m
$= \sqrt{\frac{(\sec \theta - 1)^2}{(\sec^2 \theta - 1)}} + \sqrt{\frac{(\sec \theta + 1)^2}{(\sec^2 \theta - 1)}}$	½ m
$= \sqrt{\frac{(\sec \theta - 1)^2}{\tan^2 \theta}} + \sqrt{\frac{(\sec \theta + 1)^2}{\tan^2 \theta}}$ $= \frac{(\sec \theta - 1)}{\sin^2 \theta} + \frac{(\sec \theta + 1)}{\sin^2 \theta}$	
$= \frac{(\sec \theta - 1 + \sec \theta + 1)}{\tan \theta}$	½ m
$= \frac{(2\cos\theta)}{\cos\theta\sin\theta}$	
$= \frac{1}{\sin \theta}$ = R.H.S	½ m
Hence proved	

Q.25. A line intersects y-axis and x-axis at point P and Q, respectively. If R(2,5) is the mid-point of line segment PQ, then find the coordinates of P and Q.

The coordinates of the mid-point of the line segment joining the point are

$$[(x_1 + x_2)/2, (y_1 + y_2)/2]$$

Consider the coordinates of P as (x, y) and Q as (x_2, y_2)

The midpoint of PQ = (2, -5)

Using the midpoint formula

$$x = (x_1 + x_2)/2$$
 and $y = (y_1 + y_2)/2$

$$2 = (x_1 + x_2)/2$$
 and $-5 = (y_1 + y_2)/2$

By cross multiplication

$$\mathbf{x}_1 + \mathbf{x}_2 = \mathbf{4}$$

$$y_1 + y_2 = -10$$

As the line PQ intersects the Y-axis at P

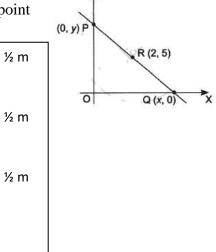
$$x_1 = 0$$

In the same way,
$$y_2 = 0$$

$$x_2 = 4$$
 and $y_1 = -10$

So the coordinates of P is (0, -10) and Q is (4, 0).

Therefore, the coordinates of P and Q are (0, -10) and (4, 0).



½ m

SECTION C

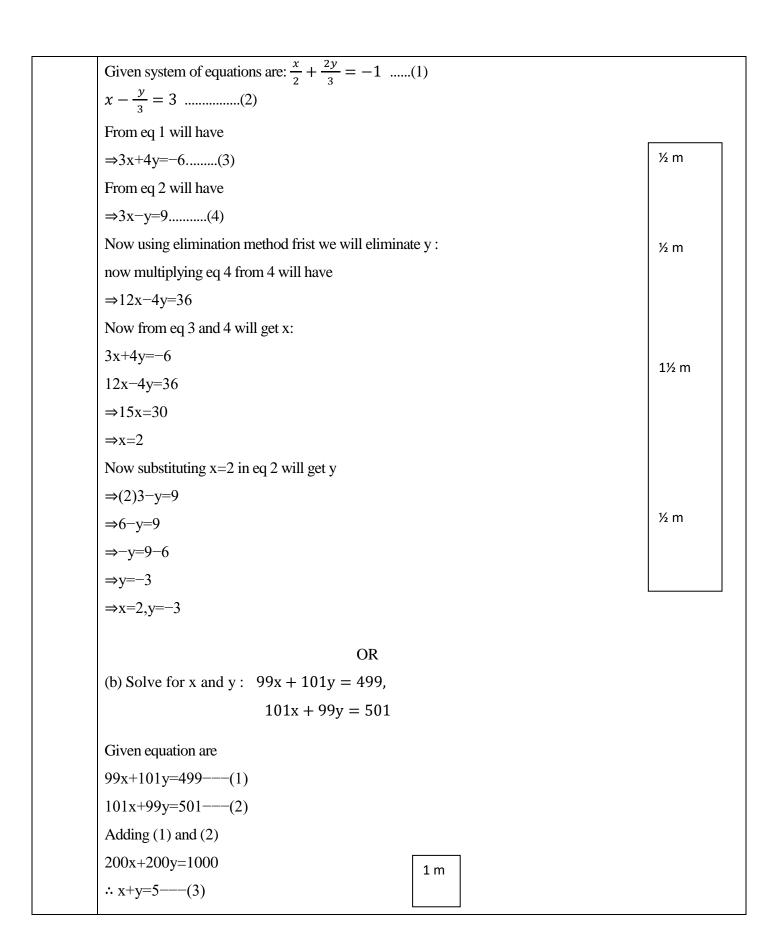
This section comprises of short answer (SA) type questions of 3 marks each.

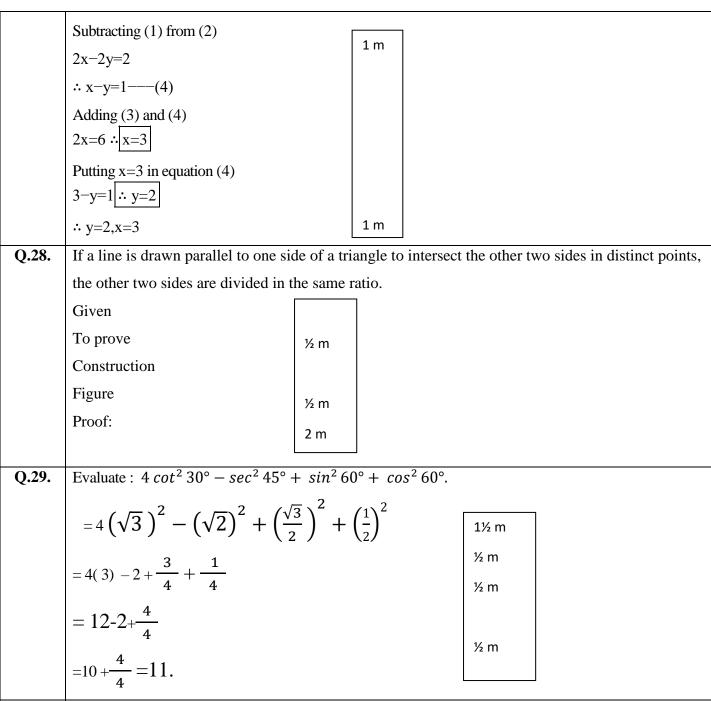
Q.26. Find the zeroes of the quadratic polynomial $5x^2 - 8x - 4$ and verify the relationship between the zeroes and the coefficient of the polynomial.

Let
$$f(x) = 5x^2 - 8x - 4$$

By splitting the middle term, we get

	$f(x) = 5x^2 - 10x + 2x - 4$	
	= 5x(x-2) + 2(x-2)	
	= (5x + 2)(x - 2)	½ m
	On putting $f(x) = 0$ we get	½ m
	(5x + 2)(x - 2) = 0	
	$\Rightarrow 5x + 2 = 0 \text{ or } x - 2 = 0$	
	x = -2/5 or x = 2	½ m
	Thus, the zeroes of the given polynomial $5x^2 - 8x - 4$ are -2/5 and 2	
	<u>Verification</u>	
	Sum of zeroes = $\alpha + \beta = \frac{-2}{5} + 2$	½ m
	$= \frac{-2+10}{5} = \frac{8}{5} \text{ or}$ $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{(-8)}{5} = \frac{8}{5}$	½ m
	Product of zeroes = $\alpha\beta = \frac{-2}{5} \times 2 = \frac{-4}{5}$ or	½ m
	$= \frac{\text{Constant term}}{\text{Coefficient of } \mathbf{x}^2} = \frac{-4}{5}$ Hence verified	
Q.27.	(a) Solve for x and y:	
	$\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$	





Q.30.	Find the value of 'm	' from the following da	ta, if its mode is 48.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	m	18	15	8

Given mode = $48 \Rightarrow \text{Modal class} = 40-50$	½ m
So, f0=12, f1=m, f2=18, l=40 and h=10	½ m
\Rightarrow Mode= l+ $\left(\frac{f1-f0}{2f1-f0-f2}\right) \times h$	½ m
$\Rightarrow 48 = 40 + \left(\frac{m - 12}{2m - 12 - 18}\right) \times 10$	
$\Rightarrow 48-40 = \left(\frac{m-12}{2m-30}\right) \times 10$	½ m
$\Rightarrow \frac{8}{10} = \frac{m-12}{2m-30}$	
$\Rightarrow \frac{4}{5} = \frac{m-12}{2m-30}$	½ m
5 2 <i>m</i> -30	
$\Rightarrow 8m -120 = 5 \text{ m} -60$ $3m = 120$	
	½ m
∴ m=40	

Q.31. a) If α and β be the zeroes of polynomial $x^2 + x - 6$, then find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

 α and β are the zeroes of the polynomial

So,
$$\alpha + \beta = -1$$
 and $\alpha\beta = -6$
Now, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2}$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2 \beta^2}$$

$$= \frac{(-1)^2 - 2(-6)}{(-6)^2}$$

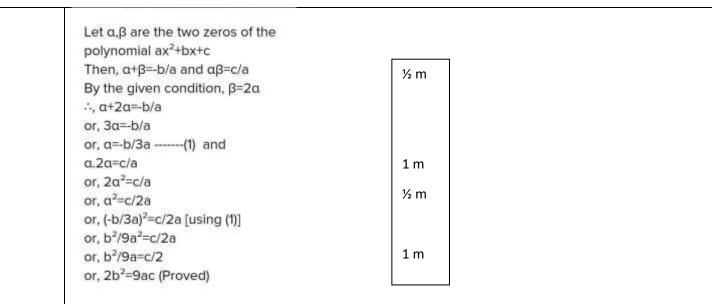
$$= \frac{1 + 12}{36}$$

$$= \frac{13}{36}$$

½ m
1 m
½ m

OR

b) If one zero of the polynomial $ax^2 + bx + c$ is double of the other, then show that $2b^2 = 9ac$.



SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

Q.32. (a) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction..

Let the fraction $\frac{x}{y}$ ½ m According to given question, $\frac{x+2}{y+2} = \frac{9}{11}$ 11(x+2) = 9(y+2)1 m 11x + 22 = 9y + 1811x - 9y + 22 - 18 = 0 $11x - 9y + 4 = 0 \dots (1)$ Again According to given question, ½ m $\frac{x+3}{y+3} = \frac{5}{6}$ 6(x+3) = 5(y+3)6x + 18 = 5y + 156x - 5y + 18 - 15 = 01 m $6x - 5y + 3 = 0 \dots (2)$

Multiply (1) with 6 and (2) with 11

66x - 54y = -24

66x - 55y = -33y = 9sub y=9 in (2)1½ m 6x-45+3=06x = 42X=7½ m Therefore fraction = 7/9OR (b) Find the value of 'p' for which the quadratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0; p \neq -1$ has real and equal roots. The given quadratic equation is $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0$; $p \ne -1$ Compare given equation with the general form of quadratic equation, ½ m which ax²+bx+c a = (p+1), b = -6(p+1) and c = 3(p+9)½ m For equal roots, $D = b^2 - 4ac = 0$ $36(p+1)^2 - 4(p+1) \times 3(p+9) = 0$ ½ m or, $3(p^2 + 2p + 1) - (p + 1)(p + 9) = 0$ ½ m or, $3p^2 + 6p + 3 - (p^2 + 9p + p + 9) = 0$ $2p^2 - 4p - 6 = 0$ or, 1 m $p^2 - 2p - 3 = 0$ or, or, $p^2 - 3p + p - 3 = 0$ p(p-3) + 1(p-3) = 0(p-3)(p+1)=0or,

or $x^2 - 6x + 9 = 0$ or, (x-3)(x-3) = 0

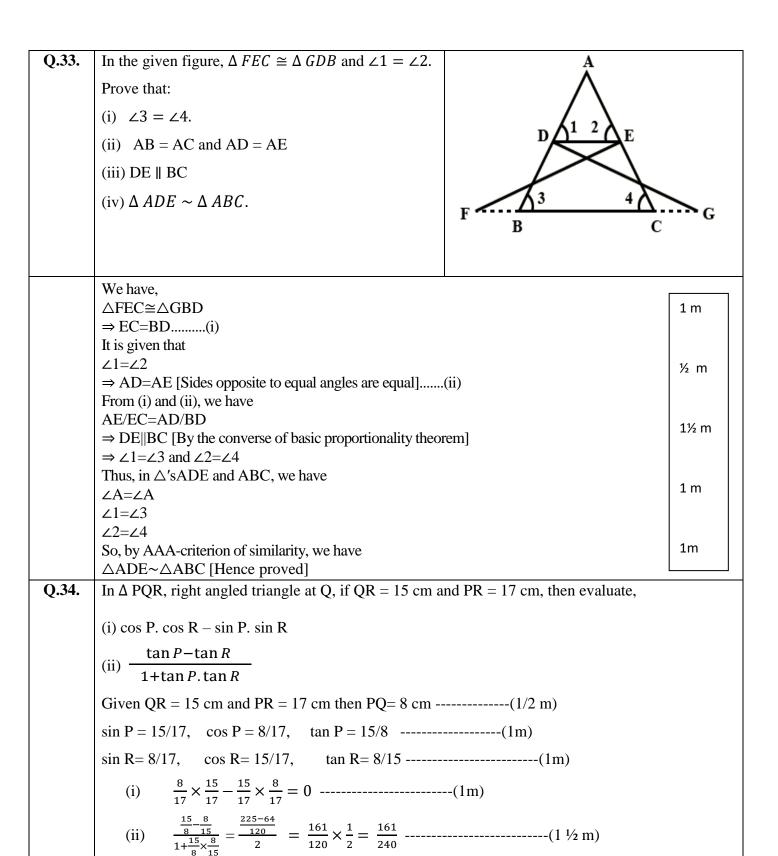
p = (m + 1) + (m) = 1

Neglecting $p \neq -1$: p = 3Now the equation becomes $4x^2 - 24x + 36 = 0$

 \therefore roots are = 3.3

½ m

1 ½ m



Q.35. (a) The following table shows the ages of the patients admitted in the hospital during a year:

Age(in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of patients	6	11	21	23	14	5

Find the mode and mean of the data given above.

Mode: The class which have highest frequency.

In this case, class interval 35-45 is the modal class.

Now, Lower limit of modal class, l=35,h=10,f1,=23,f0=21,f2=14

We know that, Mode= l+ $\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right) \times h$ =35 + $\left(\frac{23-21}{2(23)-21-14}\right) \times 10$

 $= 35 + \left(\frac{2}{11}\right) \times 10 = 35 + 1.818 = 36.8$

½ m

1½ m

Age (in years)	Mid - value	Frequency (f,)	$u_i = \frac{x_i - 30}{10}$	f_i u_i
5 – 15	10	6	-2	- 12
15 – 25	20	11	-1	- 11
25 – 35	30	21	0	0
35 – 45	40	23	1	23
45 – 55	50	14	2	28
55 - 65	60	5	3	15
Total		80		43

$$\begin{array}{l} \therefore u_i = \frac{x_i - A}{h} = \frac{x_i - 30}{10} \\ \\ \overline{x} = A + h \frac{\sum f_i u_i}{\sum f_i} = 30 + 10 \times \frac{43}{80} \\ = 30 + 5.375 \\ = 35.375 \\ \approx 35.37 \end{array}$$

Calcu 1½ m

OR

(b) The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class Interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20

600-700	y
700-800	9
800-900	7
900-1000	4

Hence, x = 9and y = 15.

We have,

Class interval	Frequency (f)	Cumulative frequency (cf)
0-100	2	2
100-200	5	7
200-300	x	7+x
300-400	12	19+x
400-500	17	36+x
500-600	20	56+x
600-700	у	56+x+y
700-800	9	65+x+y
800-900	7	72+x+y
900-1000	4	76+x+y
		Total = 100

N = ∑f₁ = 100
⇒ 76 + x + y = 100 ⇒ x + y = 24
It is given that the median is 525. Clearly, it lies in the class 500 – 600
∴ 1 = 500, h = 100, f = 20, F = 36 + x and N = 100
Now,
Median = i +
$$\frac{\frac{N}{2}F}{f}$$
 × h
⇒ 525 = 500 + $\frac{30-(36\pi x)}{20}$ × 100
⇒ 525 – 500 = $(14 - x)$ × 5
⇒ 25 = 70 – 5x ⇒ 5x = 45 ⇒ x = 9
Putting x = 9 inx + y = 24, we get y = 15.

Table 2 m

1 m

½ m

1 ½ m

	SECTION E				
	This section comprises 3 case study- based questions of 4 marks each.				
Q.36.	Case Study- 1 A book seller has 420 Science stream books and 180 Arts stream books. He wants to stack them in such a way that each stack has the same number and they take up the least area of the surface. Based on the above information, answer the following questions:				
	(i)	Find the prime factorisation of 420 and 180. $420 = 2^2 \times 3 \times 7 \times 5$ ½ m $180 = 2^2 \times 5 \times 3^2$ ½ m	1m		
	(ii)	What is the sum of exponents of prime factors in the prime factorisation of 540? $540=7 \times 3^2 \times 2 \times 5$ the sum of exponents = 5	1m		
	(iii)	(a) What is the maximum number of books that can be placed in each stack for this purpose? 420= 2 ² x 3 x 7x 5	2m		
		$180 = 2^{2} \times 5 \times 3^{2}$ HCF(420,180) (½ m) $= 2^{2} \times 5 \times 3 = 60$ (1½ m) OR	2m		
	(b) The book seller later found that he has not placed 540 fiction books. If the				
		book seller stacks them in the similar way, then find the maximum number of books that can be placed in each stack.			
		$540=7 \times 3^2 \times 2 \times 5$			
		$420= 2^2 \times 3 \times 7 \times 5$ $180= = 2^2 \times 5 \times 3^2$ HCF (420,180,540) (½ m)			
0.27		=2 x3x5=30 (1 ½ m)			

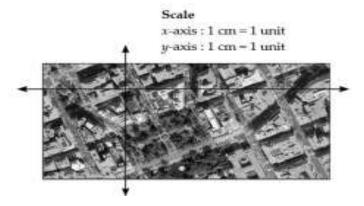
Q.37. Case Study- 2: A part of monthly hostel charges in a college is fixed and the remaining depends on the number of days one has taken food in the mess. When a student Anu takes food for 25 days, she has to pay $\stackrel{?}{<}$ 4500 as hostel charges, whereas another student Renu who takes food for 30 days, has to pay $\stackrel{?}{<}$ 5200 as hostel charges. Considering the fixed charges per month be $\stackrel{?}{<}$ x and the cost of food per day be $\stackrel{?}{<}$ y.



Based	d on the above information, answer the following questions:	
(i)	Represent algebraically the situation faced by both Anu and Renu.	1m
	X+25y=4500 (½ m)	
	X+30y= 5200 (½ m)	
(ii)	Find the number of solution/solutions represented by above situations in the	1m
	system of linear equations.	
	$\frac{a1}{a2} \neq \frac{b1}{b2}$ intersecting lines (½ m)	
	One solution (½ m)	
(iii)	(a) Find the cost of food per day and the fixed charges per month for the hostel. solving the equa (1 m)	2m
	fixed charges=₹ 1000	2m
	cost of food per day =₹140 (1 m)	
	OR	
	(b) If Renu takes food for 20 days, then what is the amount she has to pay? fixed charges=₹ 1000	
	cost of food per day =₹140 (1 ½ m)	
	the amount she has to pay= 1000+20(140)=3800(½ m)	

Q.38. Case Study- 3

Satellite image of a colony is shown below. In this view, a particular house is pointed out by a flag, which is situated at the point of intersection of x and y-axes. If we go 2 cm east and 3 cm north from the house, then we reach to a Grocery store. If we go 4 cm west and 6 cm south from the house, then we reach to an Electrician's shop. If we go 6 cm east and 8 cm south from the house, then we reach to a food cart. If we go 6 cm west and 8 cm north from the house, then we reach to a bus stand.



Based on the above information, answer the following questions.

(i)	Find the coordinates of grocery store, food cart and bus stand.	
	Grocery =(2,3) food cart (6, -8) bus stand(-6,8)(1 m)	
(ii)	(a) Find the distance between grocery store and food cart.	2m
	Grocery = $(2,3)$ food cart $(6, -8)$	
	Distance formula (½ m)	
	Distance= $\sqrt{137}$ cm(1½ m)	2m
	OR	
	(b) Find the distance of the bus stand from the house.	
	bus stand(-6,8), House (0,0)	
	Distance formula (½ m)	
	Distance = $10 \text{ cm} - (1 \frac{1}{2} \text{ m})$	
(iii)	Find the ratio of distances of house from bus stand to food cart.	1m
	bus stand(-6,8), House (0,0) food cart (6, -8)	
	Distance formula(½ m)	
	Ratio = 1:1 $(\frac{1}{2} \text{ m})$	
